POLARIS

Sampling from the Multigraph Configuration Model with Prescribed Color Assortativity

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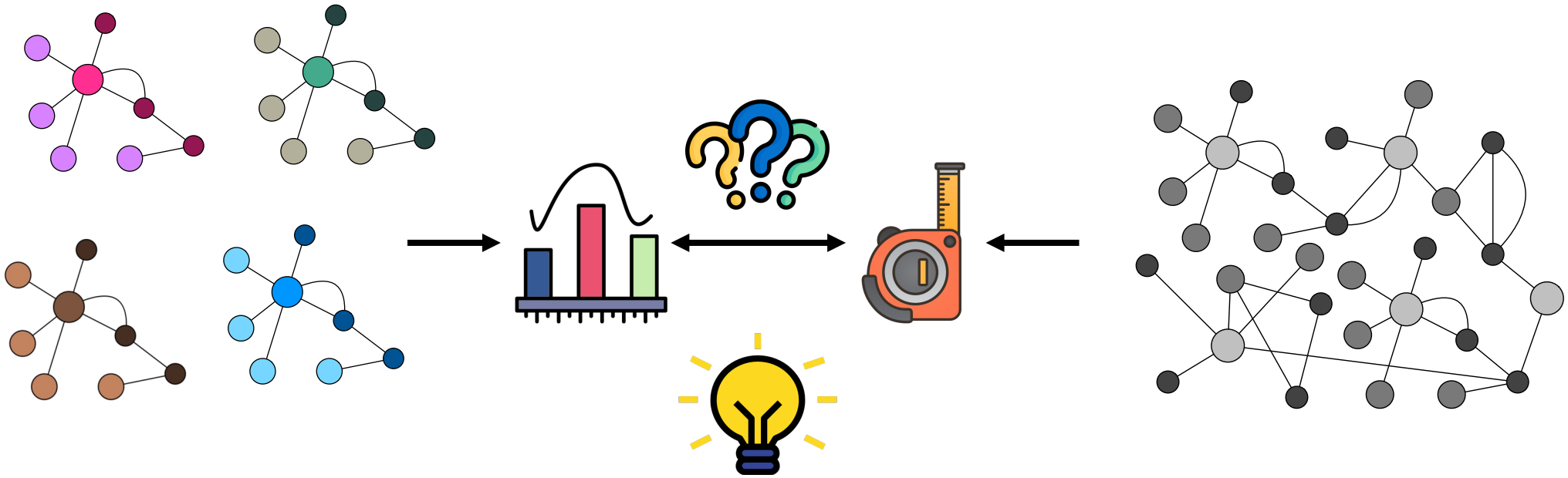
Why Graph Null Models?



we can assess
statistical significance!

What is a Null Model?

P "some" properties of the observed structure

Z structures satisfying those properties but otw random (ensemble)

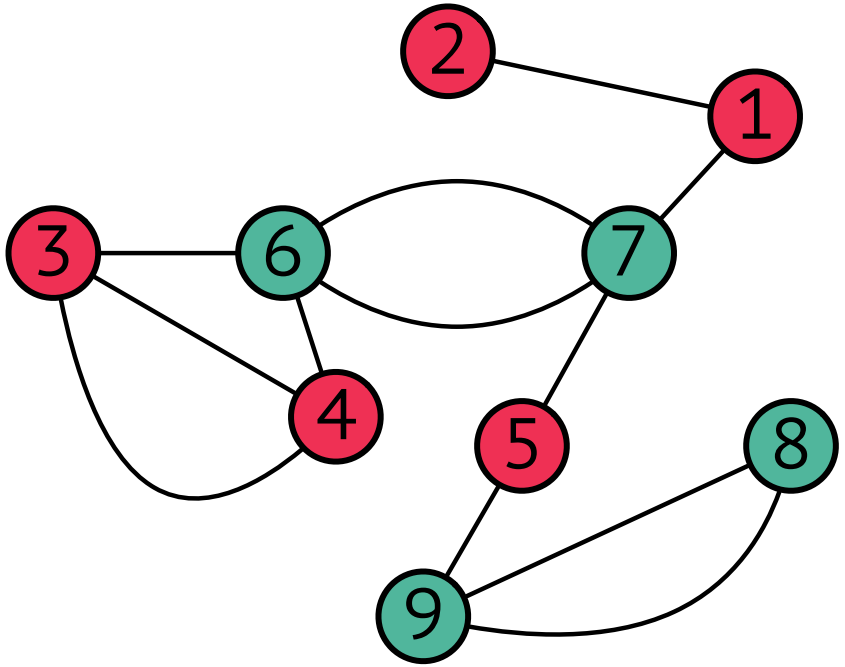
(Z, π) null model

π is a probability distribution over Z

How did we define P ?

How did we efficiently draw from Z according to π ?

Undirected Colored Multigraphs



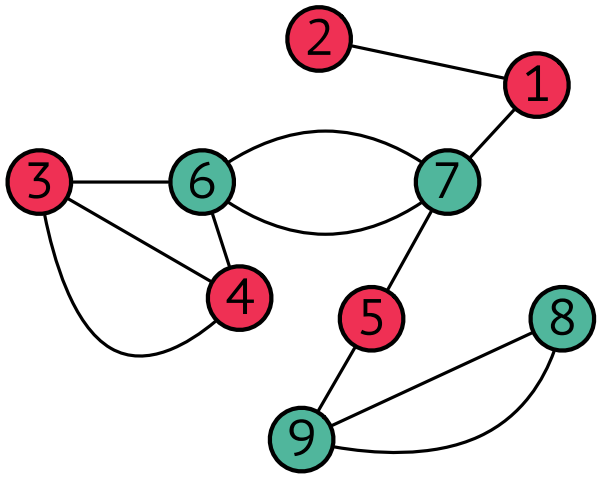
Nodes are colored

The edgeset is a multiset

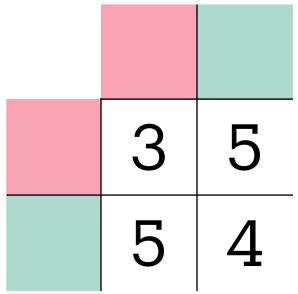
Can model social networks

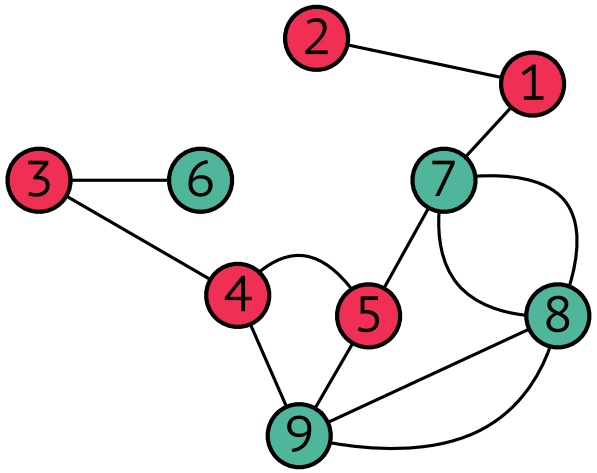
Example: Congress bill co-sponsorship network where colors are political parties

Joint Color Matrix (JCM)

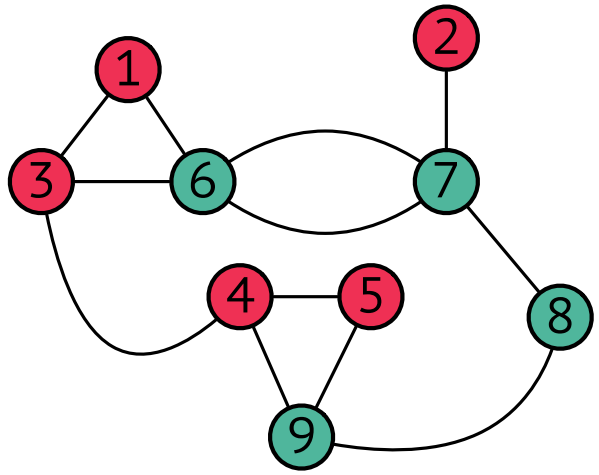


degrees: [2,1,3,3,2,4,4,2,3]
color assortativity: **0.161**





degrees: [2,1,2,3,3,1,4,4,4]
color assortativity: **0.161**



degrees: [2,1,3,3,2,4,4,2,3]
color assortativity: **0.161**

JCM preserves the color assortativity

JCM does not preserve the node degrees

Types of Ensembles

Canonical: constraints are satisfied on expectation

Micro-canonical: constraints are enforced exactly

How to sample from Micro-canonical ensembles?

Markov Chain Monte Carlo

Markov Chain Monte Carlo (MCMC)

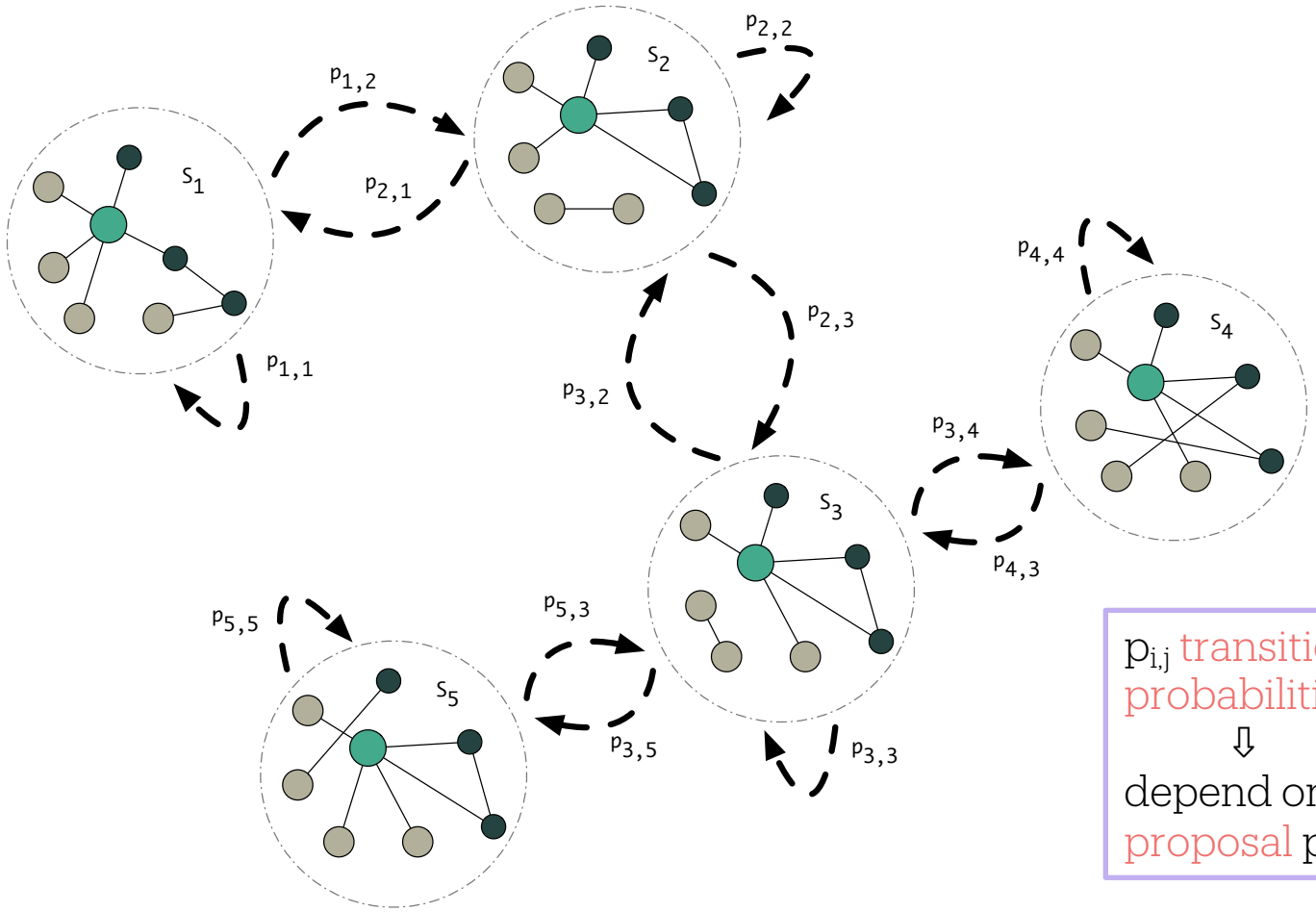
Markov Graph (MG) strongly connected and aperiodic

⇩

Markov Chain (MC) is ergodic

⇩

MC eventually samples from Z with dist. π

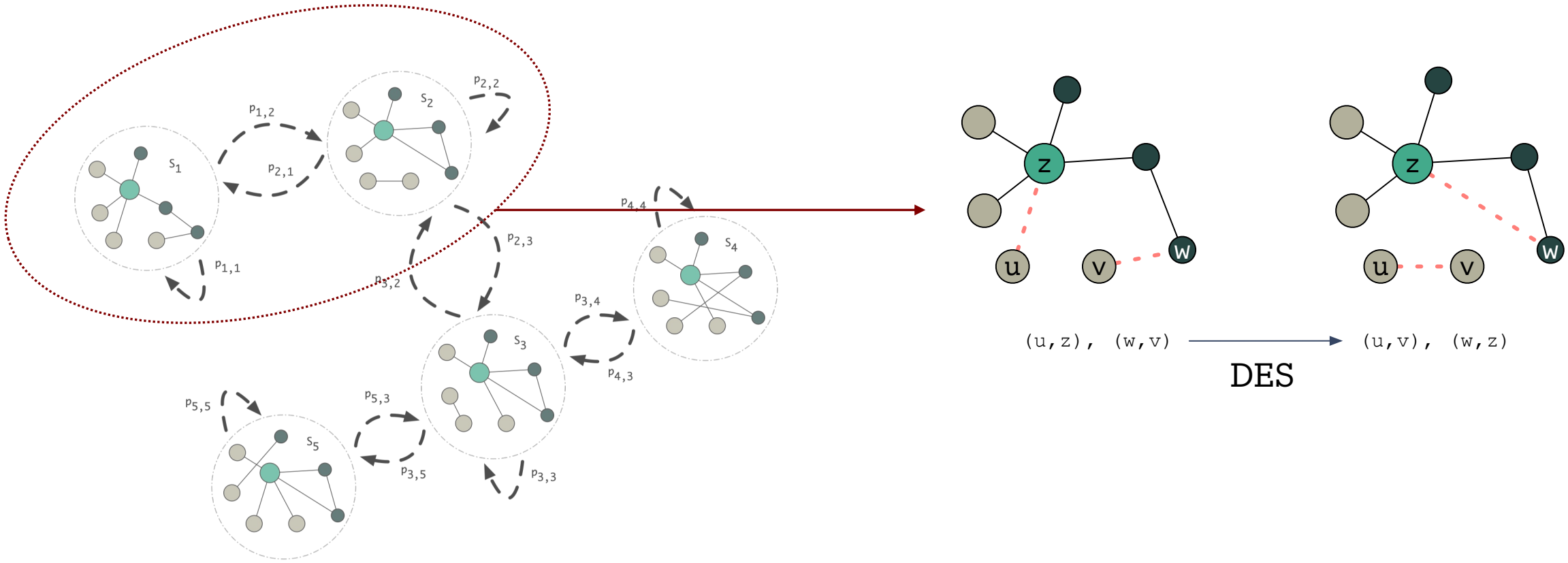


$p_{i,j}$ transition probabilities

⇩

depend on neighbor proposal probabilities

Double Edge Swaps (DESs)



Metropolis Hastings (MH)

MH randomly attempts to move onto the MG

Sometimes it accepts the move,
and sometimes it rejects it

Acceptance depends on the current state

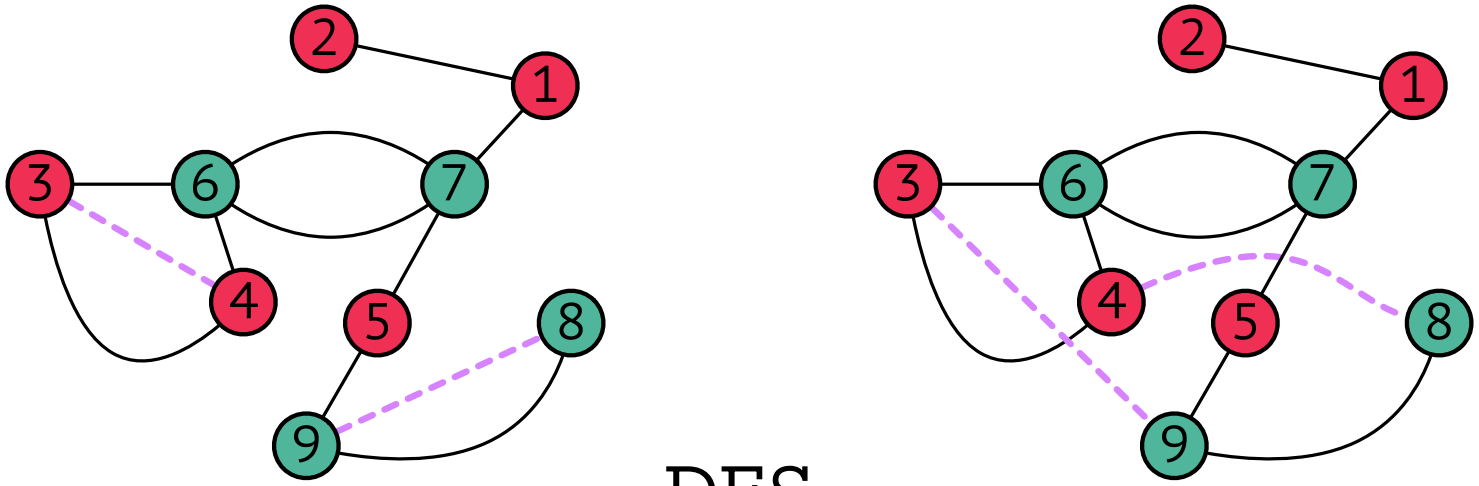
$$\alpha_v(u) = \min\left\{1, \frac{\pi(u)}{\pi(v)}\frac{\xi_u(v)}{\xi_v(u)}\right\}$$

neighbor proposal probability

A Color-agnostic Algorithm: Polaris-B

We adapted an algorithm for uncolored multigraphs

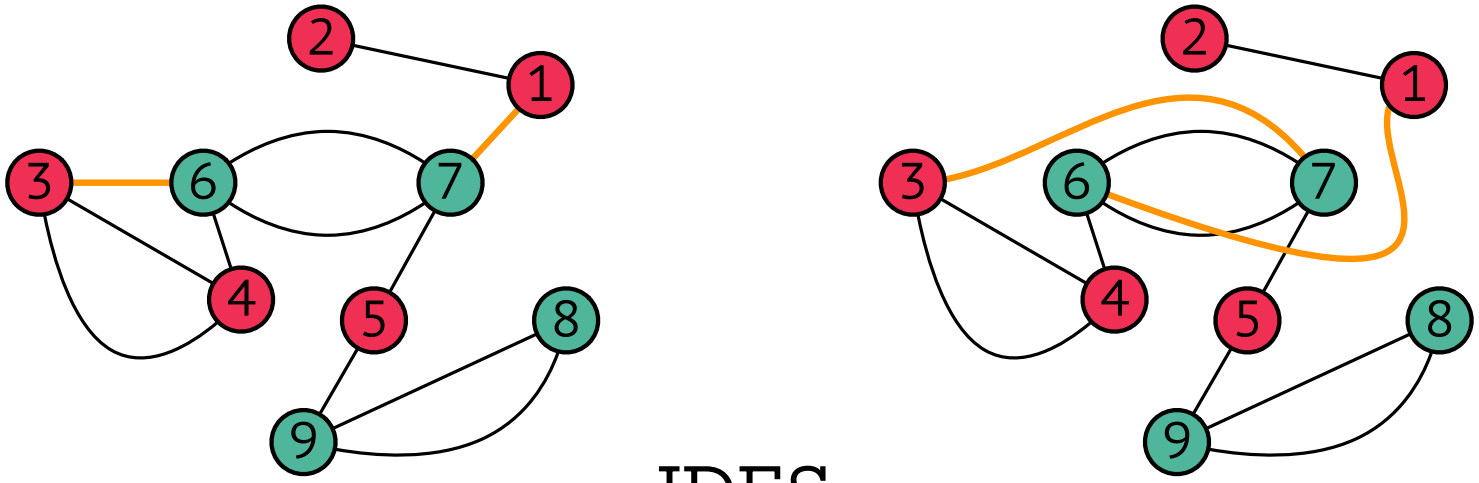
Since it operates via DESs, it must stay in the current state if the sampled DES changes the JCM



JCM-preserving Double Edge Swap (JDESs)

A JDES is a DES where the sources and/or the destinations of the edges have the same color.

This ensures that the JCM is preserved.



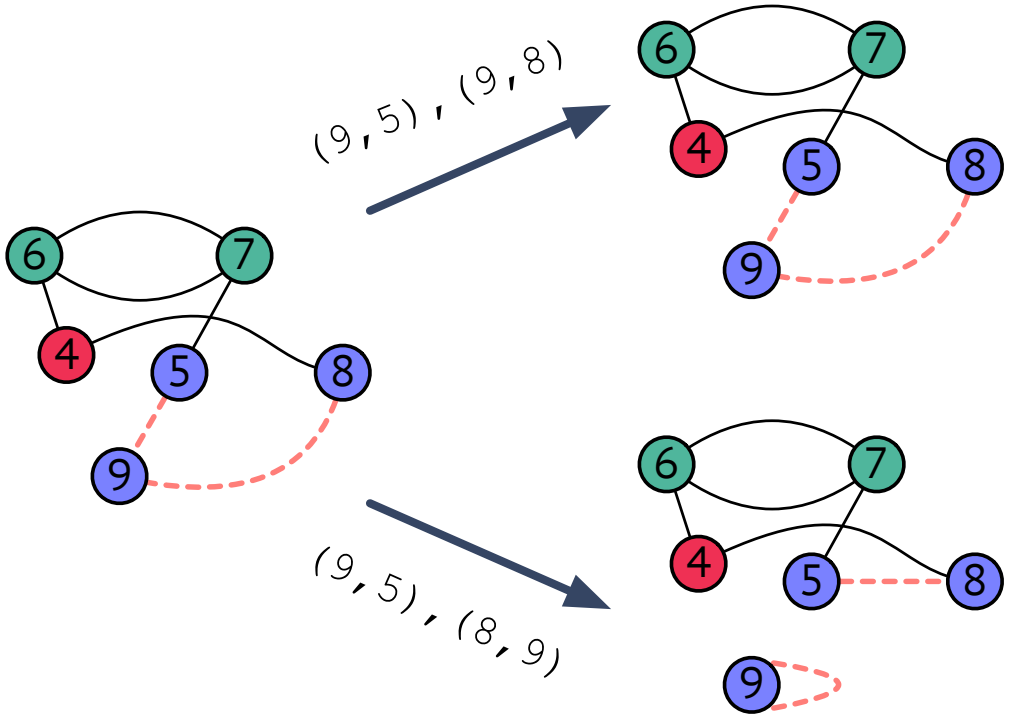
JDES

A Color-aware Algorithm: Polaris-C

Polaris-C avoids sampling pairs of edges such that neither is a JDES

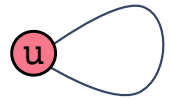
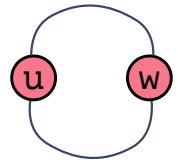
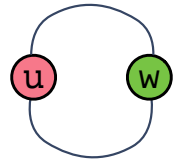
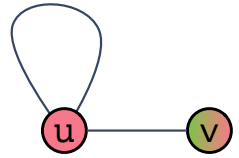
If one of the JDESs does not change the current state (no-op), it chooses the other one (*moving*)

Polaris-C reduces the probability that the MC stays in the current state



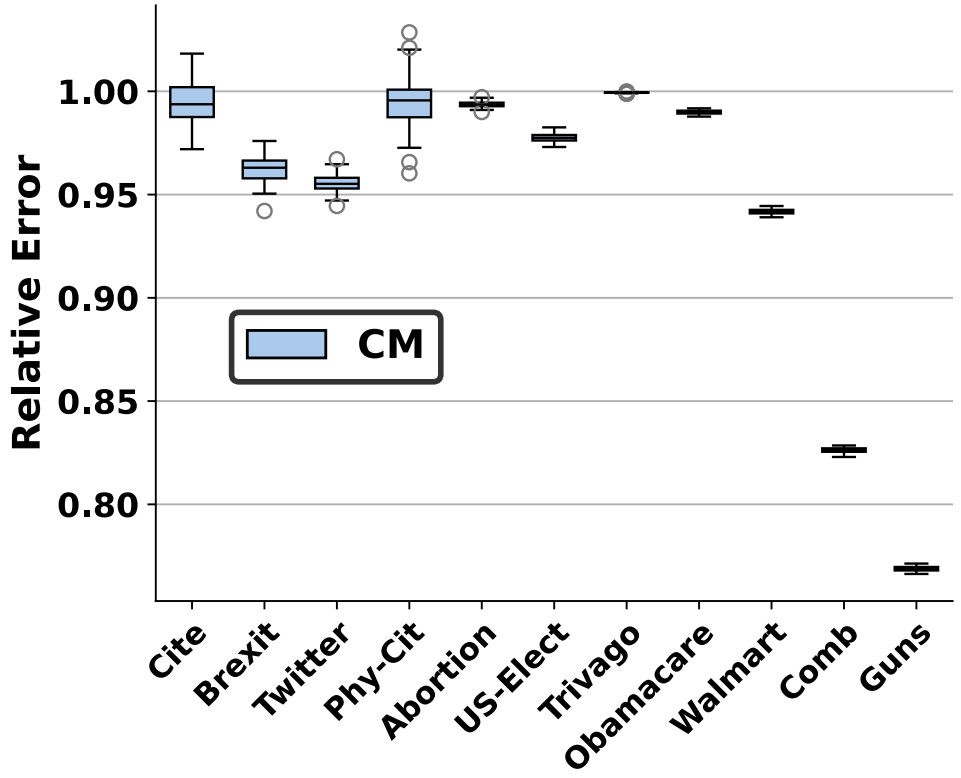
For undirected unipartite graphs, two DESs for each pair of edges!

DES Characterization

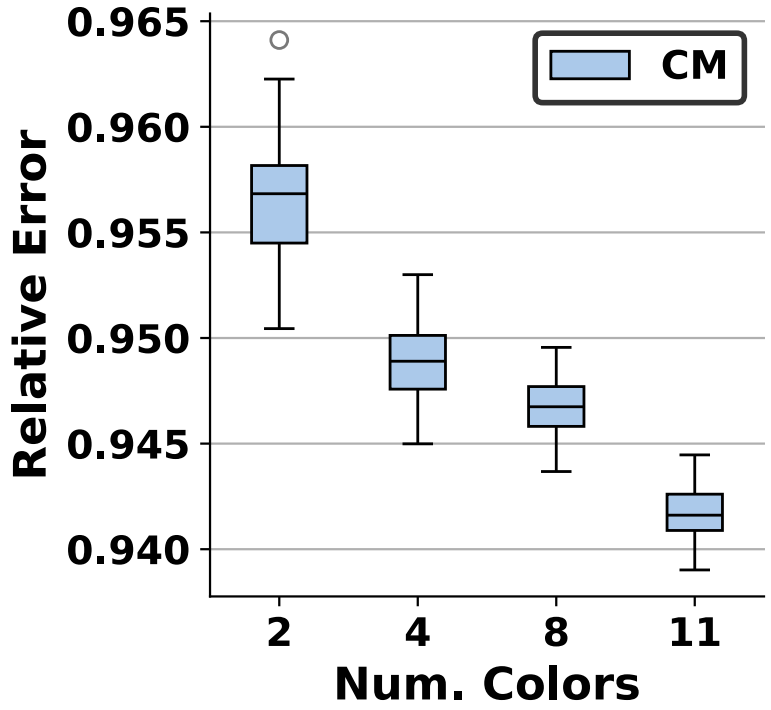
0			3b		1 moving JDES 1 no-op JDES
1		2 no-op JDESs	3c		1 moving JDES 1 no-op JDES
2a		2 moving JDESs	3d		1 no-op JDES
2b		1 moving JDES 1 no-op JDES	3e		1 no-op JDES
2c		1 no-op JDES	4a		1 moving JDES
2d		2 no-op JDESs	4b		1 moving JDES
3a		2 moving JDESs	4c		2 moving JDESs

Comparison with the Configuration Model

CM: a graph is chosen u.a.r. from the collection of all graphs with given degree sequence

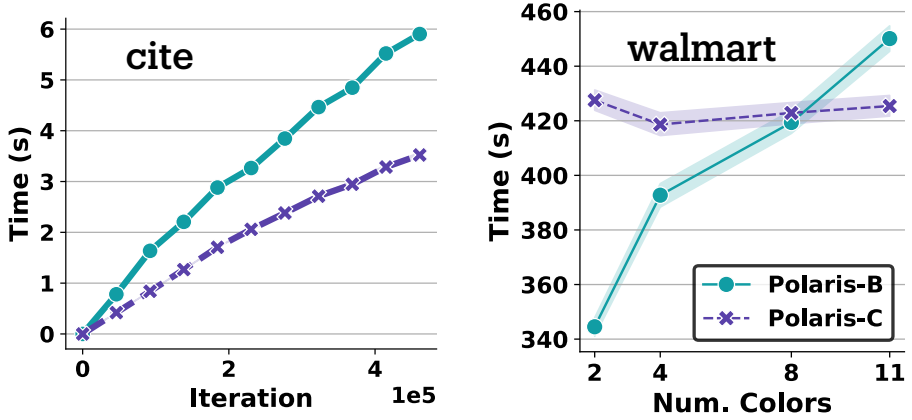


CM does not preserve color assortativity

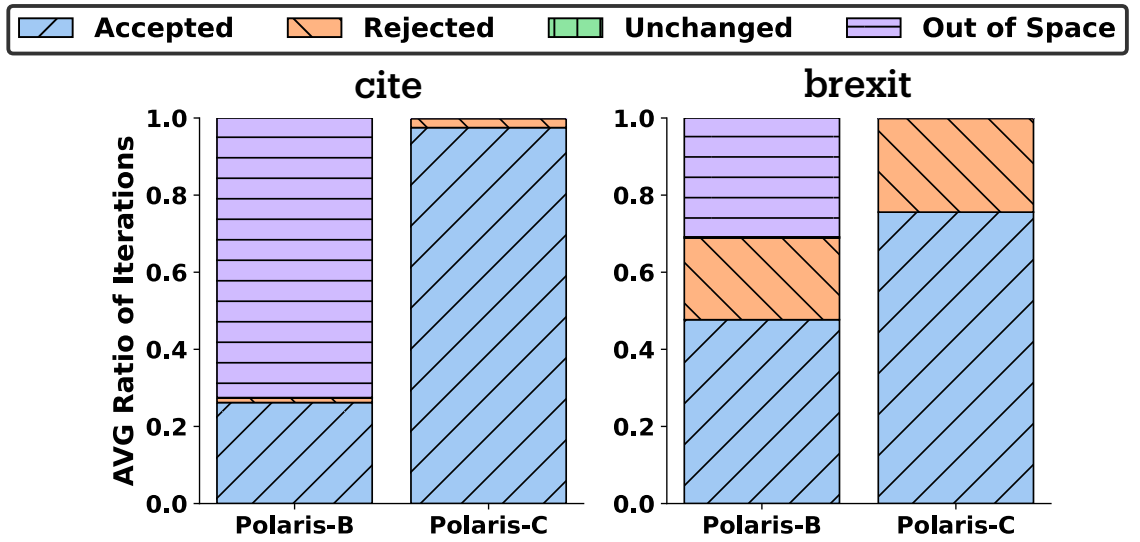


error decreases with the number of colors

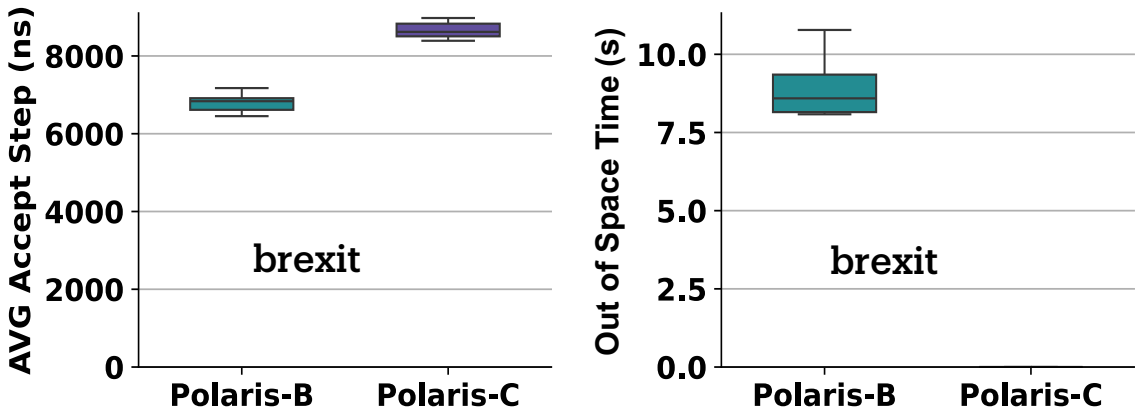
Comparison with Polaris-B



Larger numbers of colors can lead to more Out of Space operations

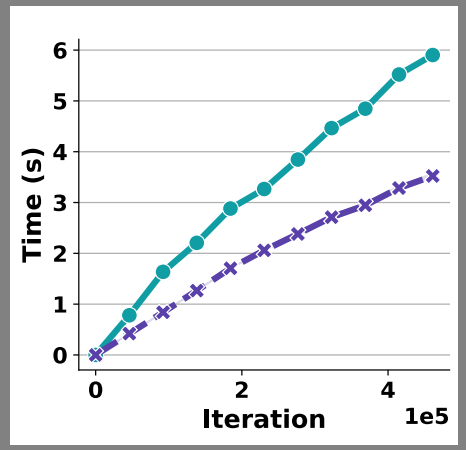


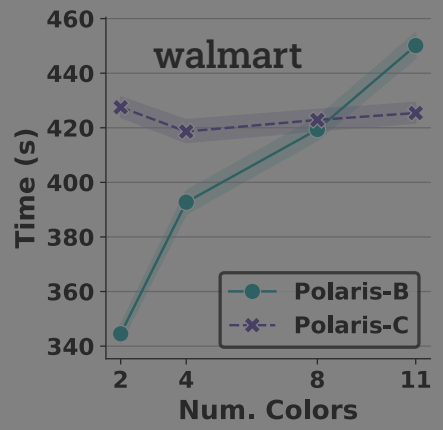
Polaris-B frequently performs self-loops



Polaris-B wastes time in Out Of Spate operations

Comparison with Polaris-B





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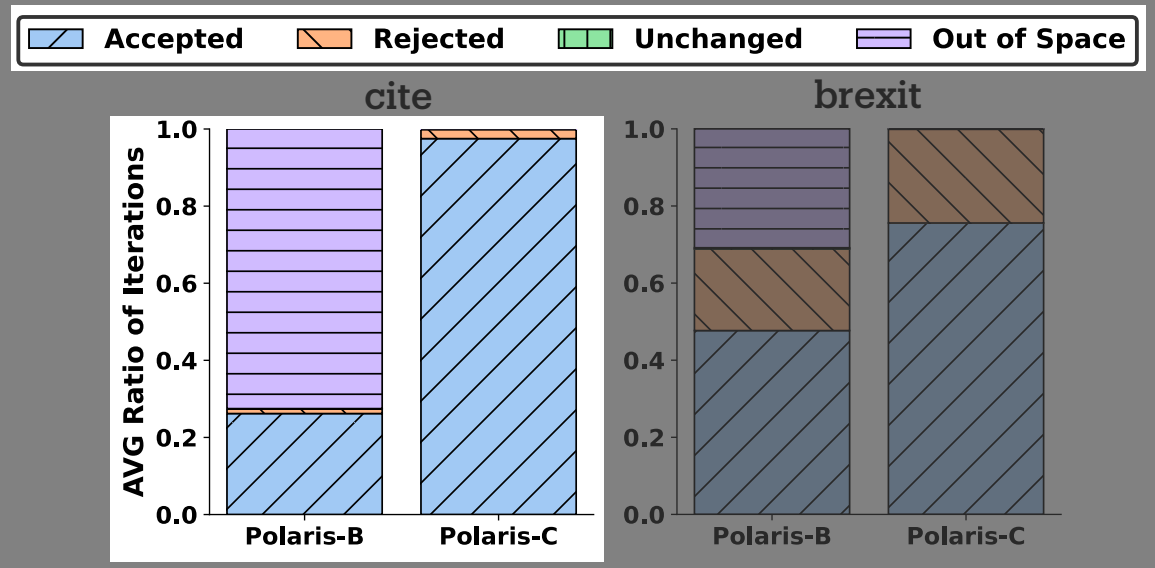

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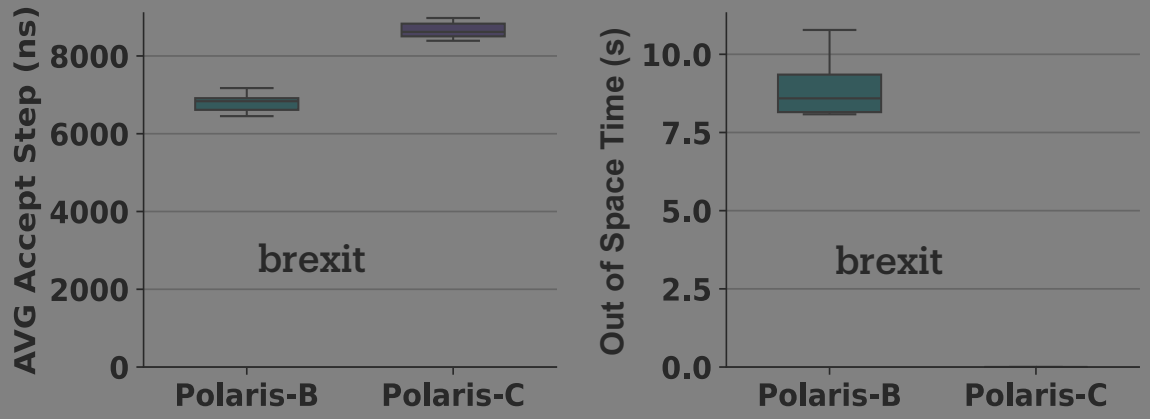

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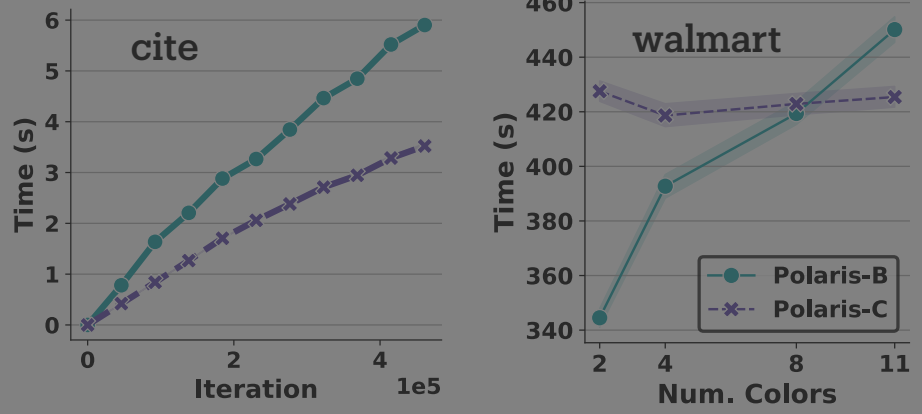

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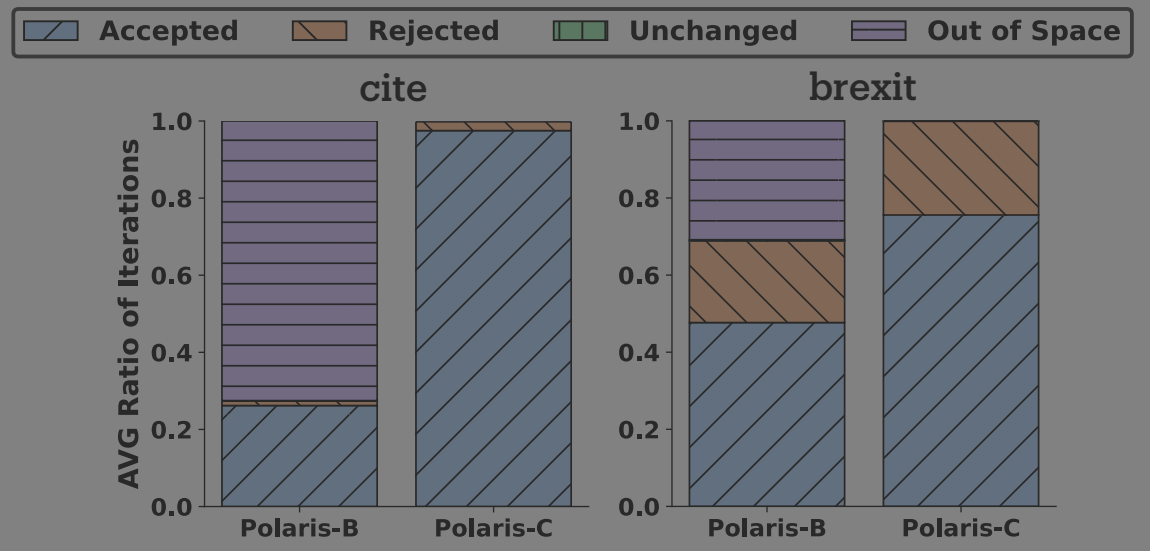

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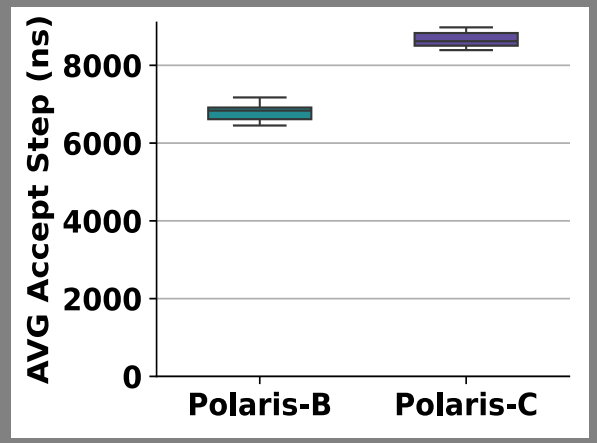

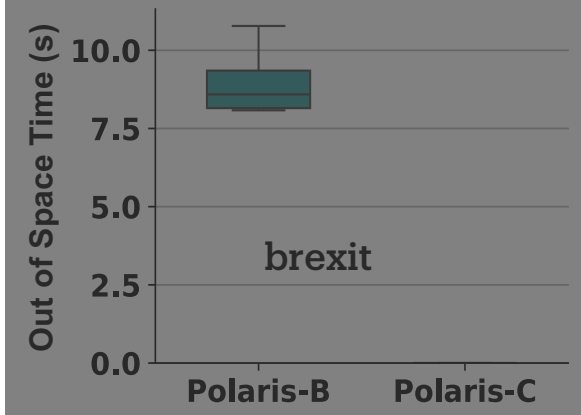

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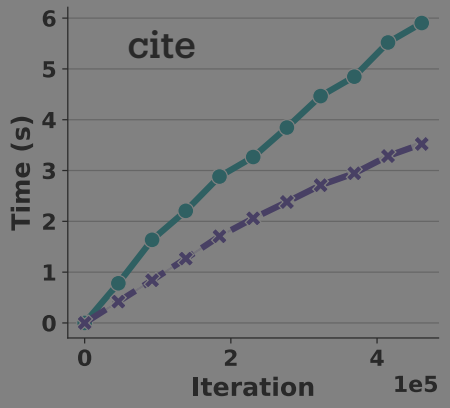

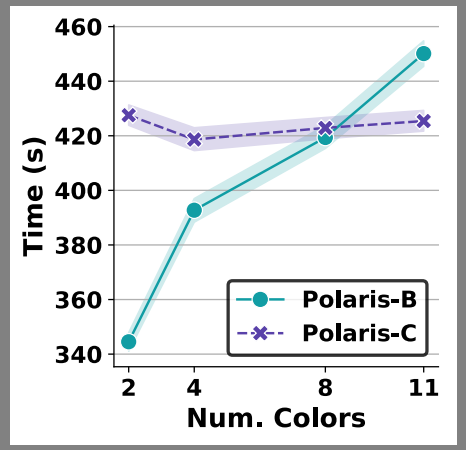

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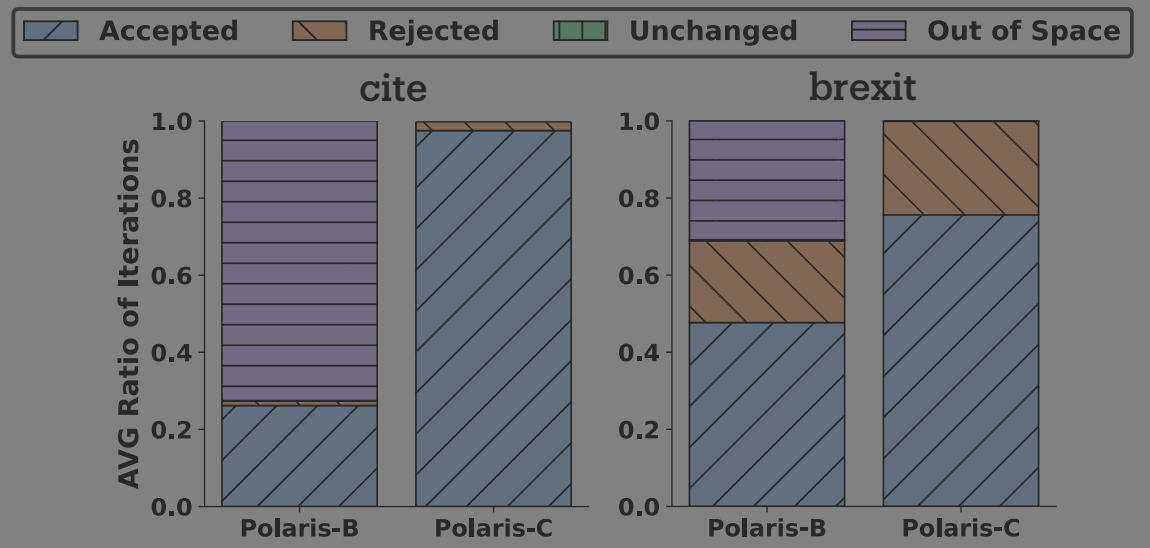

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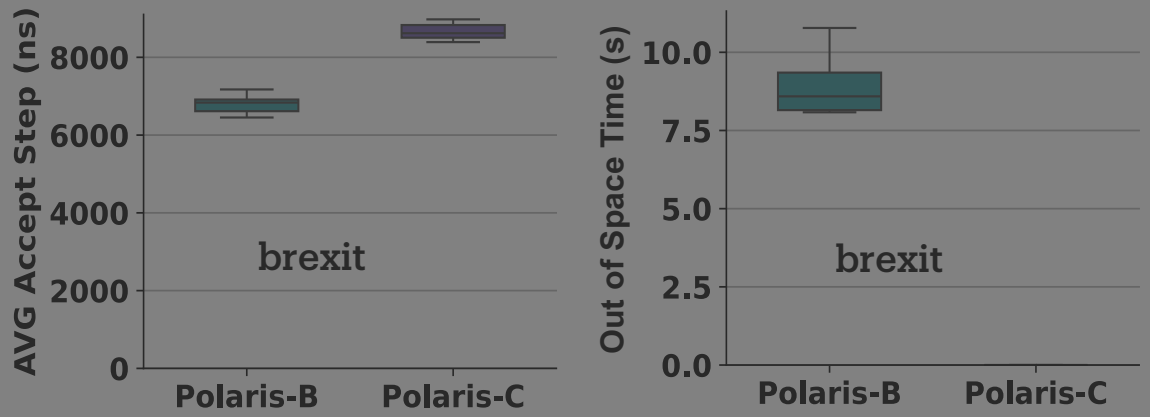

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Conclusions

- We introduced the micro-canonical ensemble of colored multigraphs with prescribed JCM
- We described two MCMC-MH sampling algorithms
- We showed the shortcomings of the CM in capturing the color assortativity
- We showed the advantages of the color-aware algorithm over the baseline
- Future works include the analysis of polarization in real networks