The Effects of Higher-Order Structures in Social Systems

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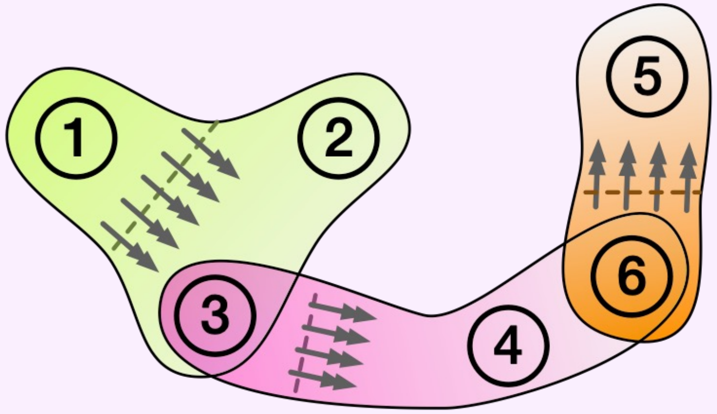
NPL RESEARCH

What is the role of structural correlations in Directed Hypergraphs?

Directed Hypergraphs

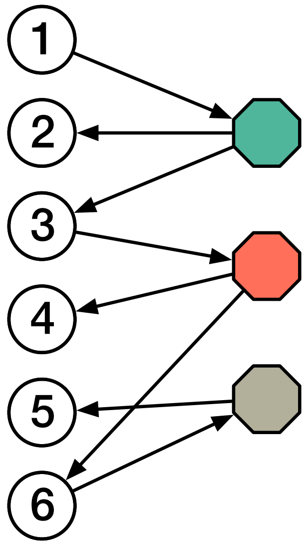


Multiset of directed hyperedges

Directed hyperedges have heads and tails

One-to-one mapping with directed bipartite graphs

Left nodes are h-vertices
Right nodes are h-edges



Why Directed Hypergraphs?

What is a Null Model?

P "some" properties of the observed structure

Z structures satisfying those properties but otw random

(ensemble)

$$(Z, \pi)$$

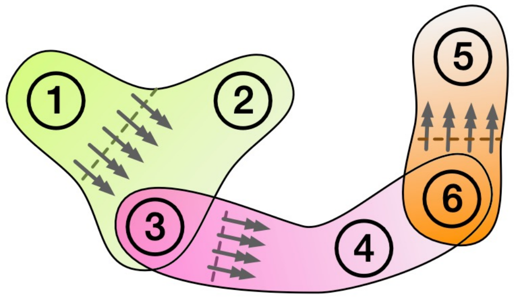
null model

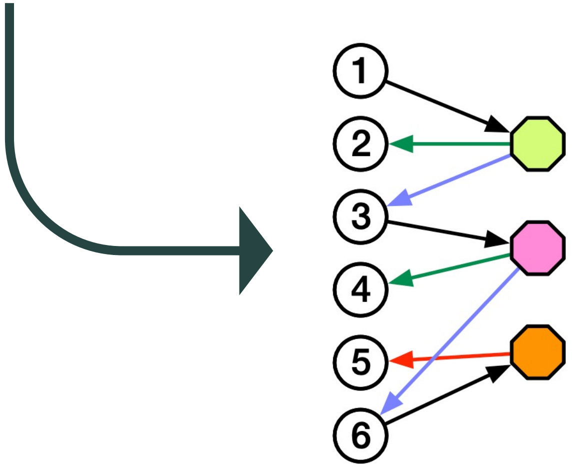
π is a probability distribution over Z

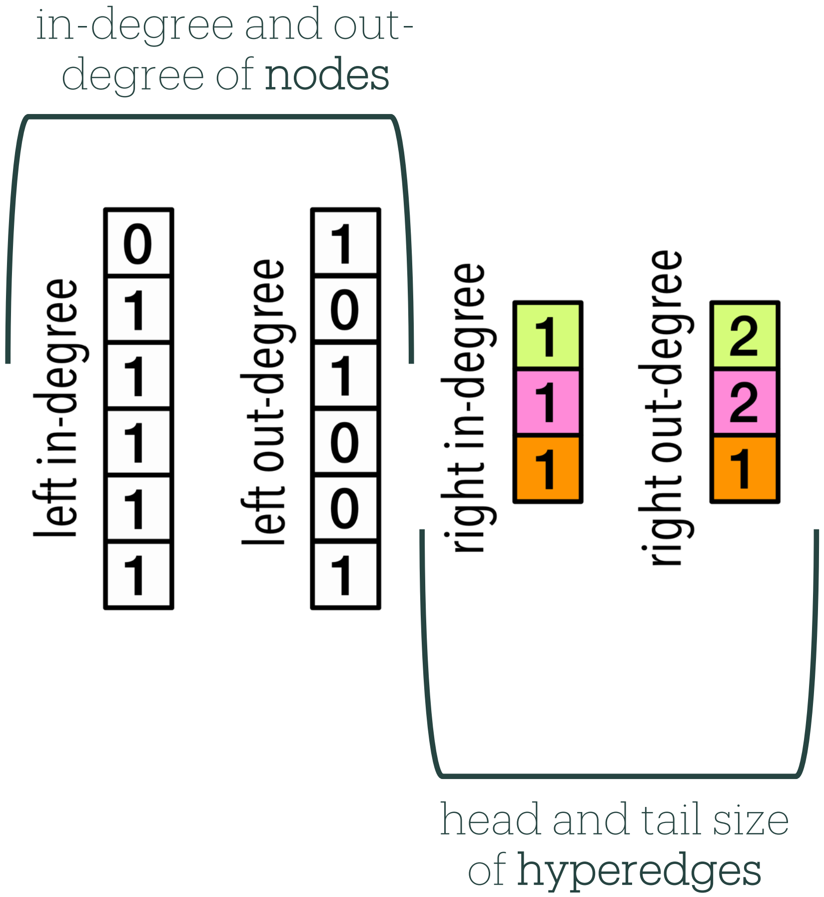
Canonical Ensemble: constraints are satisfied on expectation

Micro-canonical Ensemble: constraints are enforced exactly

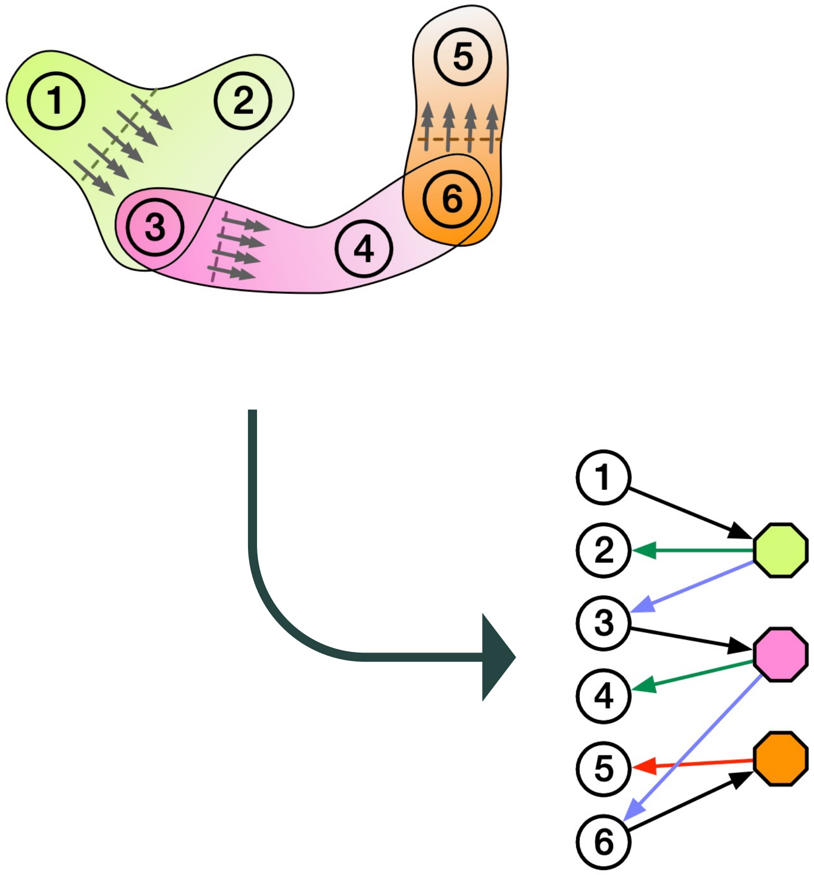
Directed Hypergraph Configuration Model





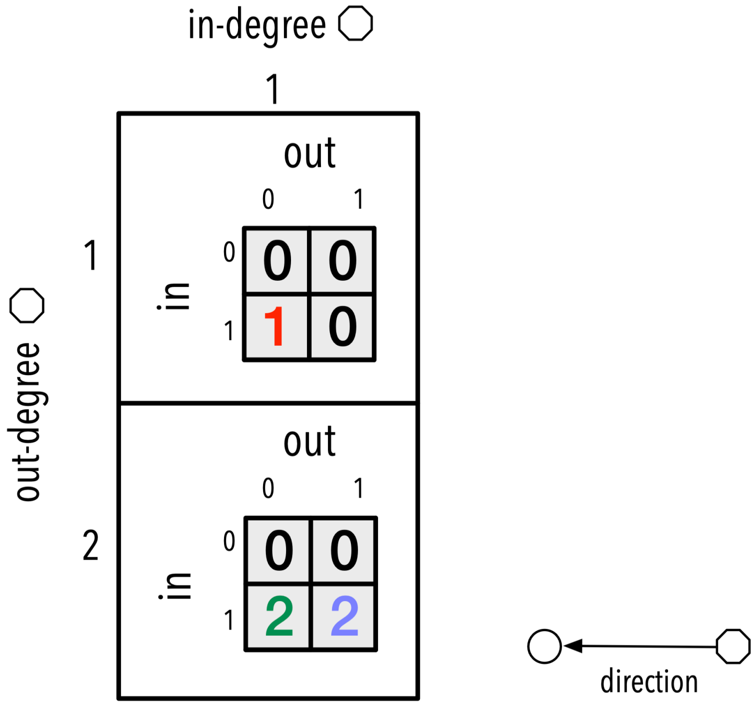


Directed JOINT Hypergraph Model



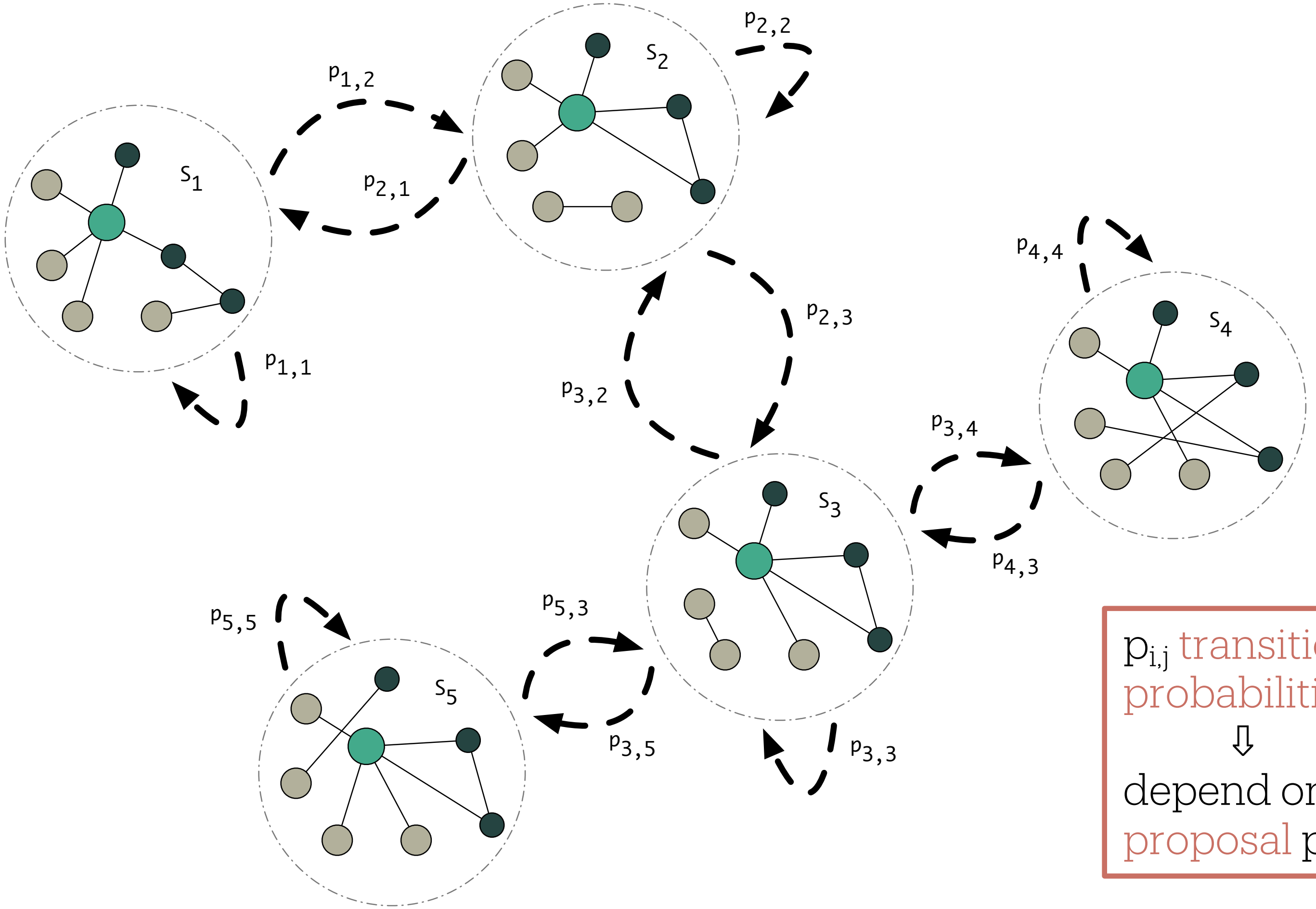
JOINT

T[i,j,k,l,d]: **number of edges** with direction d between a left vertex with in-deg i and out-deg j and a right vertex with in-deg k and out-deg l



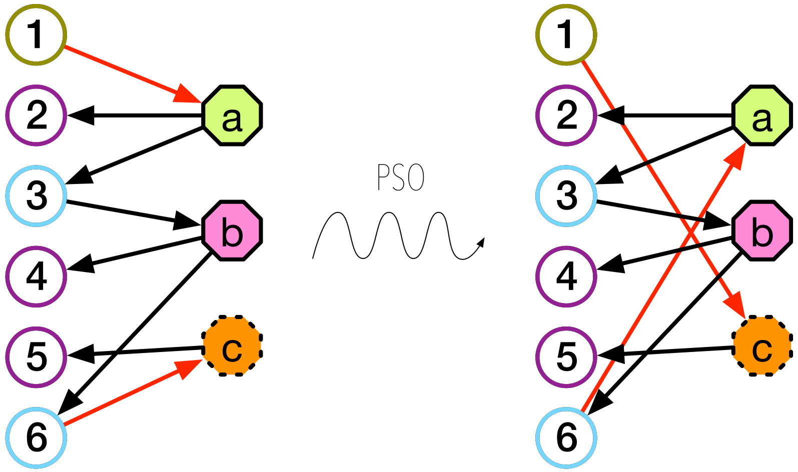
Markov Chain Monte Carlo (MCMC)

Markov Graph (MG) strongly connected and aperiodic
⇩
Markov Chain (MC) is ergodic
⇩
MC eventually samples from Z with dist. π



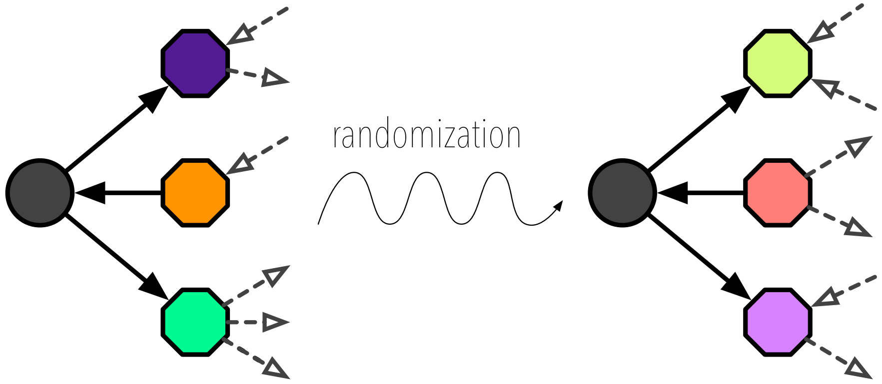
$p_{i,j}$ transition probabilities
⇩
depend on neighbor proposal probabilities

Parity Swap Operation: NuDHy-Degs

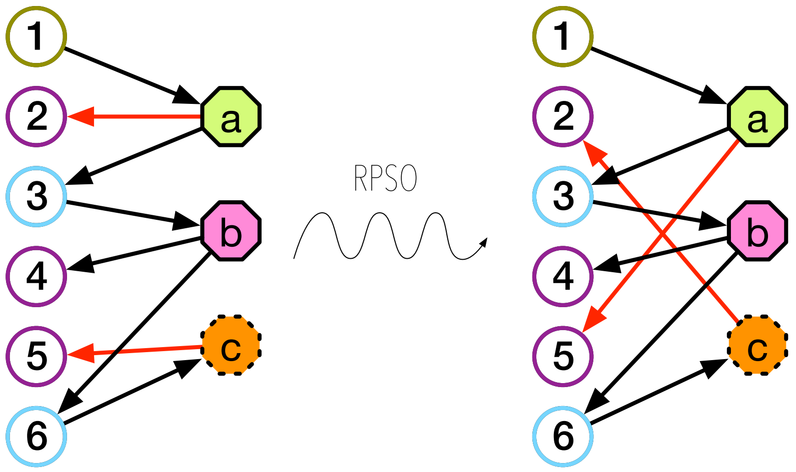


The PSO is a DES where
edges have the same direction

Changes in the Node Neighborhood

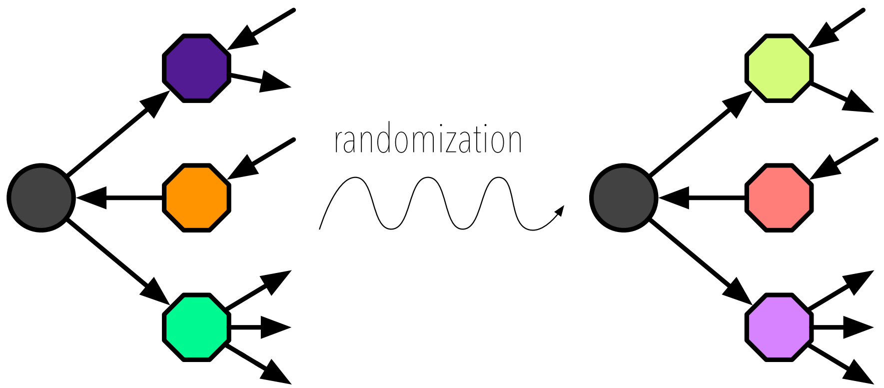


Restricted Parity Swap Operation: NuDHy-JOINT



The RPSO is a PSO where sources and/or destinations have the same in/out degrees

Changes in the Node Neighborhood



Group Affinity for Higher-Order Relations

Previous results on hypergraphs

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NETWORK SCIENCE

Combinatorial characterizations and impossibilities for higher-order homophily

Nate Veldt[1]*, Austin R. Benson[2], Jon Kleinberg[2]

num of groups with t nodes from class X

$$\mathbf{h}_t(X) = \frac{D_t(X)}{D(X)} = \frac{\sum_{v \in X} d_t(v)}{\sum_{v \in X} d(v)}$$

Affinity

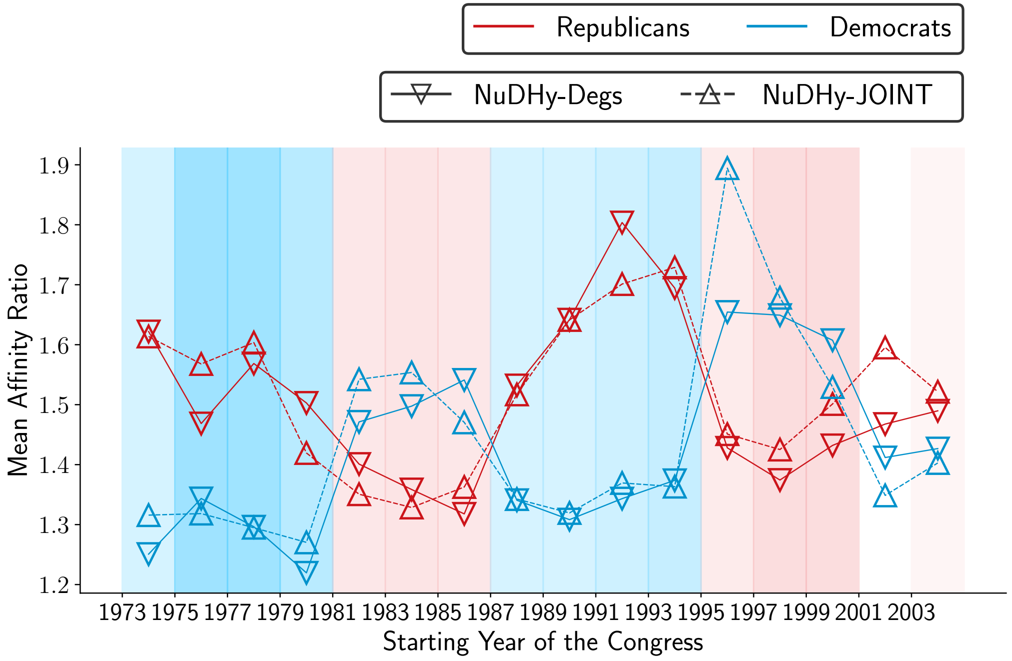
$$\hat{\mathbf{b}}_t(X) = \frac{\binom{|X|-1}{t-1}\binom{n-|X|}{k-t}}{\binom{n-1}{k-1}}$$

Baseline

extent to which entities in a certain class participate in groups with a certain number of entities from that class

null probability of participating in groups with a certain number of entities of the same class

Partisanship in US Congress Bills



When one party holds the majority of the seats the opposing party exhibits higher group affinity

“Republicans have consistently valued doctrinal purity over pragmatic deal-making”

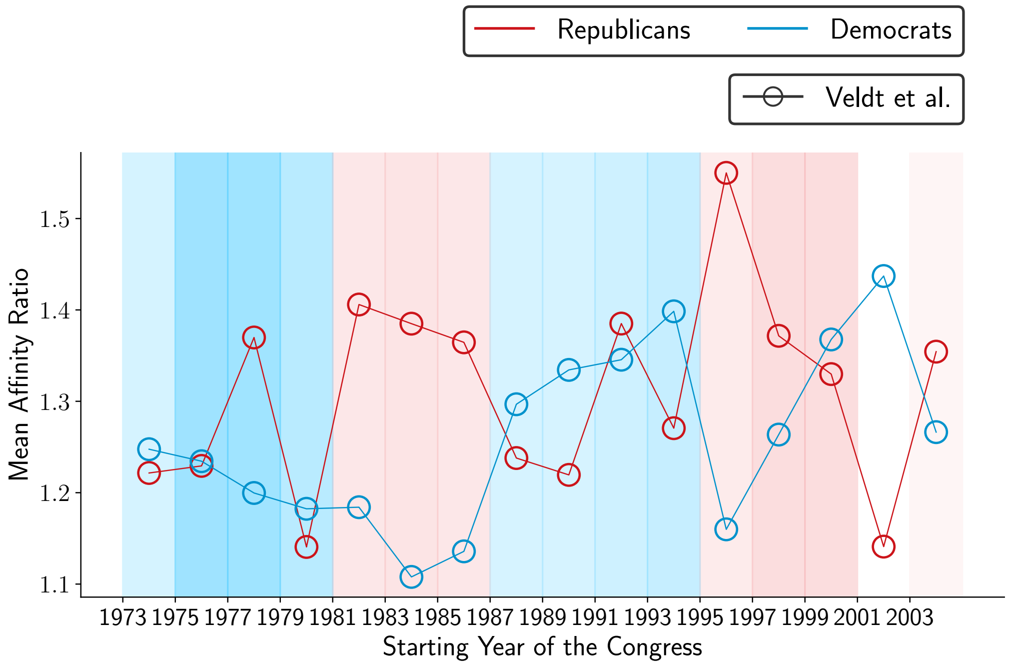
1995: Democrats display a more unified front (they sponsor fewer bills, co-sponsor intensively)

Partisanship in US Congress Bills

1995: Republicans engage in a higher rate of co-sponsorship… but also propose 2x bills!

2001: Democrats engage in a higher rate of co-sponsorship… but also propose more bills!

Baseline fails to consider each party's relative prevalence and each legislator's individual co-sponsoring opportunities



Effects on (non)-linear Dynamics

Previous results on hypergraphs

Article | Open access | Published: 06 June 2019

Simplicial models of social contagion

Iacopo Iacopini, Giovanni Petri, Alain Barrat & Vito Latora

Nature Communications **10**, Article number: 2485 (2019) | Cite this article

communications physics

ARTICLE

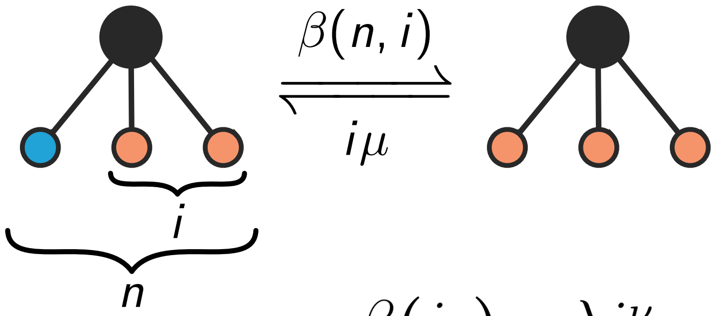
https://doi.org/10.1038/s42005-021-00788-w OPEN

Check for updates

Influential groups for seeding and sustaining nonlinear contagion in heterogeneous hypergraphs

Guillaume St-Onge[1,2✉], Iacopo Iacopini[3,4,5,6], Vito Latora[6,7,8], Alain Barrat[4,9], Giovanni Petri[10,11], Antoine Allard[1,2,12] & Laurent Hébert-Dufresne[1,12,13✉]

Hypergraph contagion

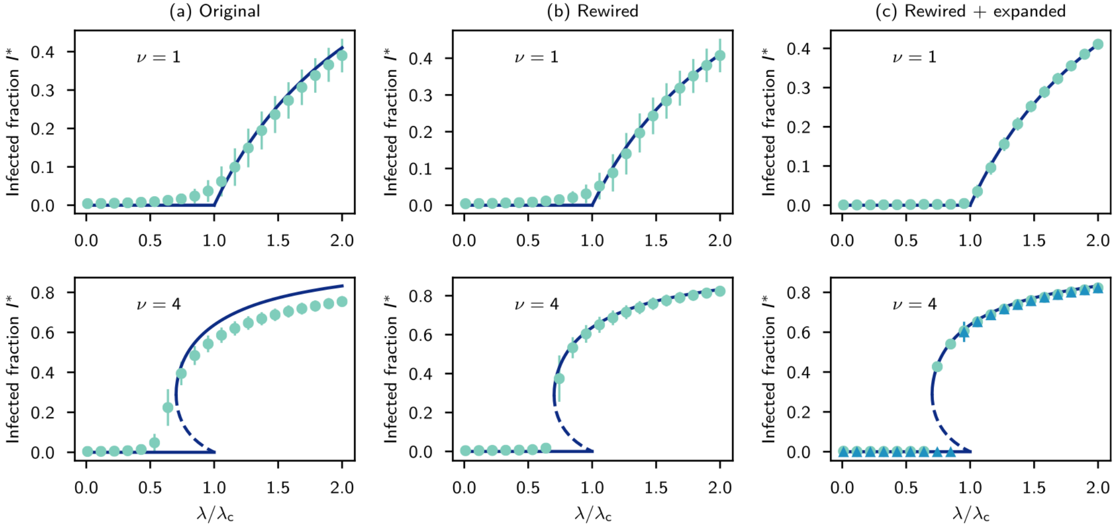


SIS model

$$\beta(i_e) = \lambda i_e^{\nu}$$

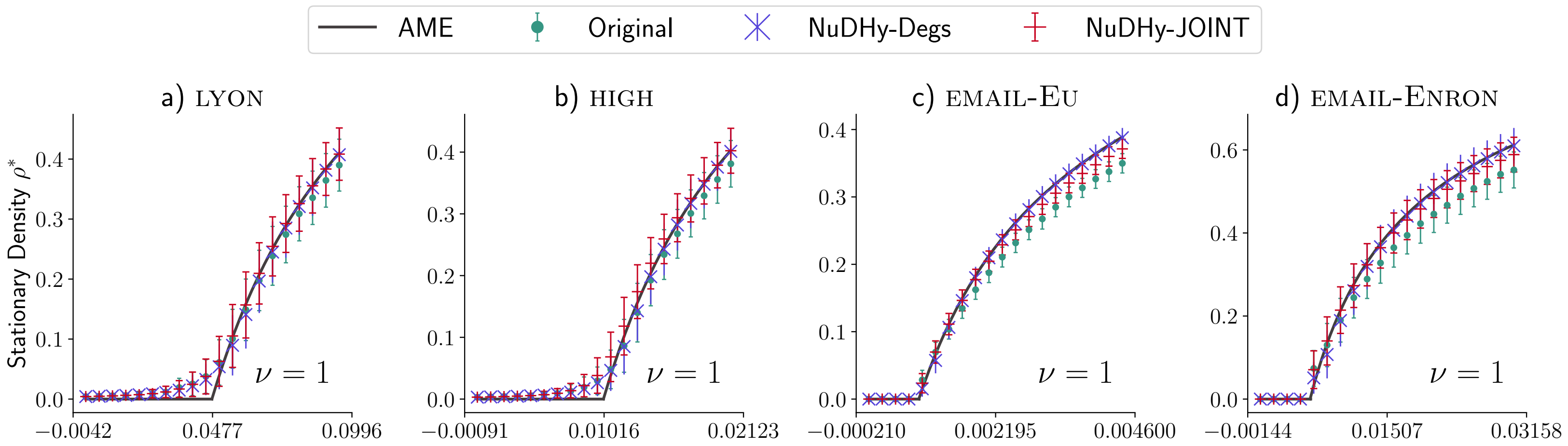
Total transition rate of v to infected $\sum_{e \in E(v)} \beta(i_e)$

contact data



Results on Contact Networks

Linear Contagions: absorbing and endemic state

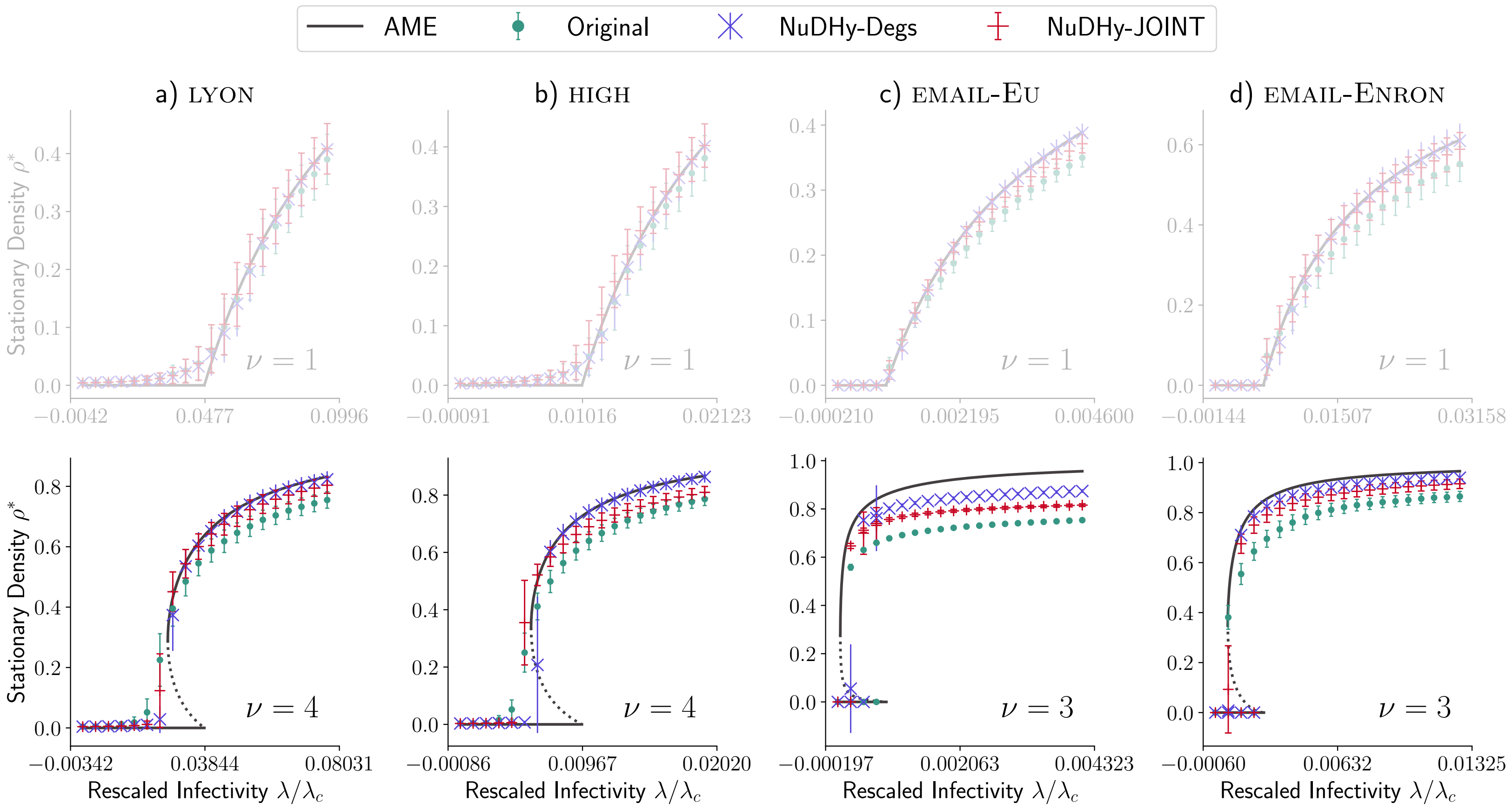


Structural correlations lead to reductions in the stationary prevalence compared to AMEs.

Results on Contact Networks

Linear Contagions: absorbing and endemic state

Super-linear Contagions: three solutions; one is unstable



Correlations especially important in the presence of nodes with large degrees.

Larger deviations in super-linear contagions and in the presence of unstable regions.

Part of the deviation can be explained by the joint degree distribution.

Resources

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